

MULTI-INDEX TOTAL TIME MINIMIZATION PROBLEM

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Abstract:

The problem of minimizing total cost of transportation problem is one of many well-structured problems in operations research that has been studied in detail. The transportation problem is one of the subclass linear programming problems which are similar in that they all require a linear function of a set of variables.

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1. INTRODUCTION

To minimize the total transportation cost is considered by many research workers in detail, however the transportation time is also relevant in a variety of real transportation problem. There are some variants of transportation problem such as minimize the total transportation time of active transportation route.

Keeping this in view and the problem review in multi index total time minimization transportation problem has been considered in which the total time is to be minimized along the active transportation routes and formulated as below.

2 . LITERATUERE SURVEY

The present work deals with multi-index time minimization transportation problem. The multi-index transportation problem is extension of classical

transportation problem where there are three indices. The method of solution is an extension of the modi-method.

There are different types of transportation problems and the simplest of them is now standard in the literature was first presented by Hitchcock (1941). It usually aims to minimize the total transportation cost. Other objectives that can be set are a minimization of the total delivery time, a maximization of profits, etc. from the investigation ;the entire existing objectives in single objective transportation model are represented by quantitative information. This may cause the negligence of some crucial points which cannot be described by quantitative data.

Later independently, by Koopman(1947).Koopman began to spearhead research on the potentialities of linear programs for the study of the problems in economics. His historic paper “optimum utilization of the transportation potations systems” was based on his war time experience. Because of this and the work done earlier by Hitchcock, the classical case is often referred as the Hitchcock-Koopman’s transportation problem. Kantorovich (1942) publishes the paper on a continuous version of the problem and later with Gavurin, an applied study of the capacitated transportation problem (Kantorovich and Gavurin 1949).

The time minimizing transportation problem has been studied by Hammer (1969),Garfinkel and Rao(1971) and Szwarc (1971).Hammer (1969) and Szwarc(1971) used labeling techniques to solve the problem . Garfunkel and Rao(1971) solved the problem by introducing a sufficiently large cost M on certain routes. Sometimes there may exist emergency situations such as those requiring police services, fire services, ambulance services, etc., when the time of transportation is of greater importance than cost of transportation. Some methods for minimizing the time of transportation have been established. Several methods



for minimizing the time of transportation are also developed. Ramakrishnan(1977) developed another method of achieving a minimum time of transportation which is very different from other existing methods. Sharma(1977) proposed a survey to provide an up-to-date account of the theoretical and computational aspects of various special cases and extensions of the transportation problem. Prakashet *al.* (1985) presented the transshipment problem with the objective to minimize the duration of transportation and to find optimal routes transportation from origins to destination with transshipment has been studied. Natarajan and Pandian (2011) presented a new blocking method for finding an optimal solution to bottleneck transportation problem, which is very different from other existing methods. They also proposed a method for finding all efficient solutions of a bottleneck-cost transportation problem. Namely, blocking zero point method is proposed which is based on zero point method. Both the proposed methods provide the necessary decision support to decision makers while they are handling time oriented logistic problems.

Nikolic(2007) proposed that the total transportation time problem regarding the time of the active transportation routes. If the multiple optimal solution exist, it is possible to include other criteria as second level of criteria and find the corresponding solutions. Furthermore, if there is multiple solution, again, the third objective can be optimized in lexicographic order. Preetvanti and P.k. (2003) presented multiple objective time transportation problem with additional restrictions(MOTTP). This MOTTP with non-linear bottleneck objective function is related to Lexicographic Multiple Time Transportation Problem With Additional Restrictions, which is solved by a lexicographic primal code. An algorithm and its

underlying theory is developed to determine an initial efficient basic solution of this MOTTP.

3. FORMULATION OF PROBLEM

The Multi-index total time minimization transportation problem in which there are m origins, n destinations and p type of commodities to be transported can be formulated as below.

$$\text{Minimize } T(x) = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} t_{ijk} h_{ijk} \quad (3.1)$$

$$h_{ijk} = \begin{cases} 1 & \text{if } x_{ijk} > 0 \\ 0 & \text{if } x_{ijk} = 0 \end{cases} \quad (3.2)$$

Subject to

$$\sum_{i=1}^n x_{ijk} = A_{jk}, \quad \sum_{j=1}^m x_{ijk} = B_{ki}, \quad \sum_{k=1}^p x_{ijk} = E_{ij},$$

Where

$$\sum_{j=1}^m A_{jk} = \sum_{i=1}^p B_{ki}, \quad \sum_{k=1}^p B_{ki} = \sum_{j=1}^m E_{ij}, \quad \sum_{i=1}^n E_{ij} = \sum_{k=1}^p A_{jk}$$

And

$$\sum_{j=1}^m \sum_{k=1}^p A_{jk} = \sum_{k=1}^p \sum_{i=1}^n B_{ki} = \sum_{i=1}^n \sum_{j=1}^m E_{ij}, \quad x_{ijk} \geq 0,$$

Here $i=1,2,\dots,m$ are the origins, $j=1,2,\dots,n$ are destinations, $k=1,2,\dots,p$ are the various type of commodities.

h_{ijk} is auxiliary function show active and non -active transportation routes (activities),

x_{ijk} is amount of type k^{th} goods sent from the i^{th} factory to the j^{th} destination ,

t_{ijk} is the time of transporting type k^{th} of goods from the i^{th} source to the j^{th} destination,

A_{jk} the requirement at the j^{th} destination of type k^{th} of goods,

B_{ki} the availability of type k^{th} of goods at the i^{th} factory ,

E_{ij} the total amount of goods to be sent from the i^{th} factory to the j^{th} destination.

This constraints show that k^{th} type of commodities received by all destination is equal to k^{th} type of commodity supplied from all origins and different types of commodities supplied by the i^{th} source is equal to amount of commodity received by all destination from the i^{th} source. The amount of commodity supplied from all origins to j^{th} destination is equal to different types of commodities received by j^{th} destination and the amount of commodities received by all destination of different types of commodities is equal to amount of commodities supplied from all origins to all destination is equal to different types of commodities supplied from all origins.

It has been assumed that the transportation starts simultaneously and the time of transportation does not depend on the amount of product transported.

4. SOLUTION PROCEDURE

In the solution procedure, we make the use of the combination of approaches given by Haley (1962) and Nolic (2007) and is given below.

Let $X^{(D)}$ and $X^{(D+1)}$ are two basis neighboring feasible solutions, where $X_{ijk}^{(D)}$ is entering basis variable and $X_{isk}^{(D)}$ is leaving basis variable for $X^{(D)}$:

$$X^{(D)} \text{ contain : } X_{ijk}^{(D)} = 0 \quad \text{and} \quad X_{isk}^{(D)} > 0$$

$$X^{(D+1)} \text{ contain : } X_{ijk}^{(D+1)} > 0 \quad \text{and} \quad X_{isk}^{(D+1)} = 0$$

$$\text{there is : } X_{ijk}^{(D+1)} = X_{isk}^{(D)}$$

In moving from $X^{(D)}$ to $X^{(D+1)}$ the total transportation time $T(x)$ of problem (3.1) will be changed with the following values:

$$q_{ijk}^{(D)} = t_{ijk} - t_{isk} \quad (3.3)$$

The characteristics q_{ijk} are the change of transportation time in problem (3.1).

Then the solution $X^{(D+1)}$ has:

$$T^{(D+1)} = T^{(D)} + (t_{ijk}^{(D)} - t_{isk}^{(D)}). \quad (3.4)$$

Clearly, the total time $T^{(D+1)}$ is determined by values $q_{ijk}^{(D)}$ as:

$$T^{(D+1)} = \begin{cases} > T^{(D)} & \text{if } q_{ijk}^{(D)} > 0 \\ = T^{(D)} & \text{if } q_{ijk}^{(D)} = 0 \\ < T^{(D)} & \text{if } q_{ijk}^{(D)} < 0 \end{cases} \quad (3.5)$$

5. NUMERICAL EXAMPLE

Consider a $3 \times 3 \times 3$ multi-index total time transportation problem.

$$\text{Minimize } T(x) = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} t_{ijk} h_{ijk} \quad (3.1)$$

$$h_{ijk} = \begin{cases} 1 & \text{if } X_{ijk} > 0 \\ 0 & \text{if } X_{ijk} = 0 \end{cases} \quad (3.2) \quad 1$$

$$\text{Subject to} \quad \sum_{i=1}^n x_{ijk} = A_{jk}, \quad \sum_{j=1}^m x_{ijk} = B_{ki}, \quad \sum_{k=1}^p x_{ijk} = E_{ij},$$

$$\text{where} \quad \sum_{j=1}^m A_{jk} = \sum_{i=1}^n B_{ki}, \quad \sum_{k=1}^p B_{ki} = \sum_{j=1}^m E_{ij}, \quad \sum_{i=1}^n E_{ij} = \sum_{k=1}^p A_{jk}$$

and
$$\sum_{j=1}^m \sum_{k=1}^p A_{jk} = \sum_{k=1}^p \sum_{i=1}^n B_{ki} = \sum_{i=1}^n \sum_{j=1}^m E_{ij}, \quad x_{ijk} \geq 0,$$

Table 3.1 gives the data-time t_{ijk} at the top left corners of the respective cells, B_{ki} at the right extreme of rows, A_{jk} at the bottom of columns and E_{ij} in the $(i, j)^{th}$ cell consisting of cells $(i, j, 1), (i, j, 2), (i, j, 3) \quad i=1, 2, 3; \quad j=1, 2, 3.$

	$j=1$	$j=2$	$j=3$	
$i=1$	6	5	7	$B_{11}=6$
	7	6	3	$B_{21}=9$
	$E_{11}=10$	6	$E_{13}=9$	$B_{31}=10$
		$E_{12}=6$	10	
			11	

$i=2$	11		9		13		$B_{12} = 13$
		8		15		7	$B_{22} = 15$
	$E_{21} = 21$	13	$E_{22} = 10$	12	$E_{23} = 14$	8	$B_{32} = 17$
$i=3$	5		8		10		$B_{13} = 15$
		6		9		6	$B_{23} = 14$
	$E_{31} = 22$		7	$E_{32} = 13$	7	$E_{33} = 12$	$B_{33} = 18$
						12	
	$A_{11} = 15$		$A_{21} = 8$		$A_{31} = 11$		
	$A_{12} = 18$		$A_{22} = 12$		$A_{32} = 8$		
	$A_{13} = 20$		$A_{23} = 9$		$A_{33} = 16$		



5. CONCLUSION

Multi –Index TOTAL TIME MINIMIZATION PROBLEM can be formulated and a procedure can be developed to find its optimal solution.

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