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Inferential Statistics

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Introduction

Statistics is a branch of mathematics used to summarize, analyze, and interpret what we observe to make sense or meaning of our observations. While analyzing data, it is possible to use both descriptive and inferential statistics.

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Descriptive statistics:

Descriptive statistics is to describe, summarize the data in a systematic way. It therefore enables us to present the data in a more meaningful way, which allows simpler interpretation of the data. Typically, the following are the techniques used in descriptive statistics.

- **Univariate analysis: Univariate analysis involves describing the distribution of a single variable, including its central tendency i.e. the mean, median, and mode and dispersion i.e. the range and quantiles of the data-set, and measures of spread like the variance and standard deviation. The shape of the distribution may also be described via indices such as skewness and kurtosis.**

- **Bivariate analysis: When a sample consists of more than one variable, descriptive statistics may be used to describe the relationship between pairs of variables. In this case, descriptive statistics include cross-tabulations and contingency tables, graphical representation via scatterplots, quantitative measures of dependence, Descriptions of conditional distributions etc.**

Inferential Statistics is a testing of hypothesis and drawing conclusions about a population, based on the sample. Inferential statistics is the branch of statistics that uses mathematical tools, of which probability theory is the main component, to draw conclusions about population statistics based on the corresponding sample statistics. If the data used in the computation comes from the whole population, then the statistic is called a population statistic. Likewise, if the data comes from a sample of the population,then the statistic is called sample statistic.

For Example: It is not practically possible to ask every Indian how they feel about fairness of voting procedures. We would sample a few thousand Indians drawn from the hundreds of millions that make up the country. But is not so easy because something would be wrong with our sample if we collect sample of people of Punjab, it could not be used to infer the attitudes of other Indians The same problem would arise if the sample were comprised the people of other state Inferential statistics are based on the assumption that sampling is random. We trust a random sample to represent different segments of society in close to the appropriate proportions. When inferential statistics, we use results obtained from a sample of individuals to make claims about a larger population.

Hypothesis Testing

The method in which we select samples to learn more about characteristics in a given population is called hypothesis testing Hypothesis testing is a method for testing a claim or hypothesis about a parameter in a population, using data measured in a sample. In this method, we test some hypothesis by determining the likelihood that a sample statistic could have been selected, if the hypothesis regarding the population parameter are true. Hypothesis is mainly of two types, the null hypothesis and the alternative hypothesis.

The null hypothes(H0)is a claim of no difference.It is a statement about a population parameter, such as the population mean, that is assumed to be true. The null hypothesis attempts to show that no variation exists between variables, or that a single variable is no different than zero. For example, if we want to investigate the effect of hormons on the growth of animals, the null hypothesis will be: the hormones on the growth of

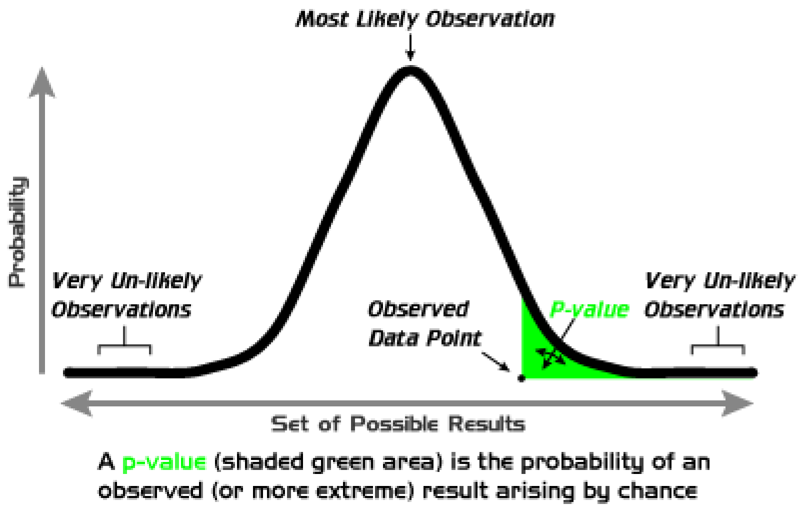
H0: There is no significant difference between the effect of the hormons on the growth of the animals.

The alternative hypothesis is a claim of a difference in the population. An alternative hypothesis(H1)is a statement that directly contradicts a null hypothesis by stating that the actual value of a population parameter is less than, greater than, or not equal to the value stated in the null hypothesis. In the above example, the alternative hypothesis will be:

Ha:There is significant difference between the effect of the hormons on the growth of the animals.

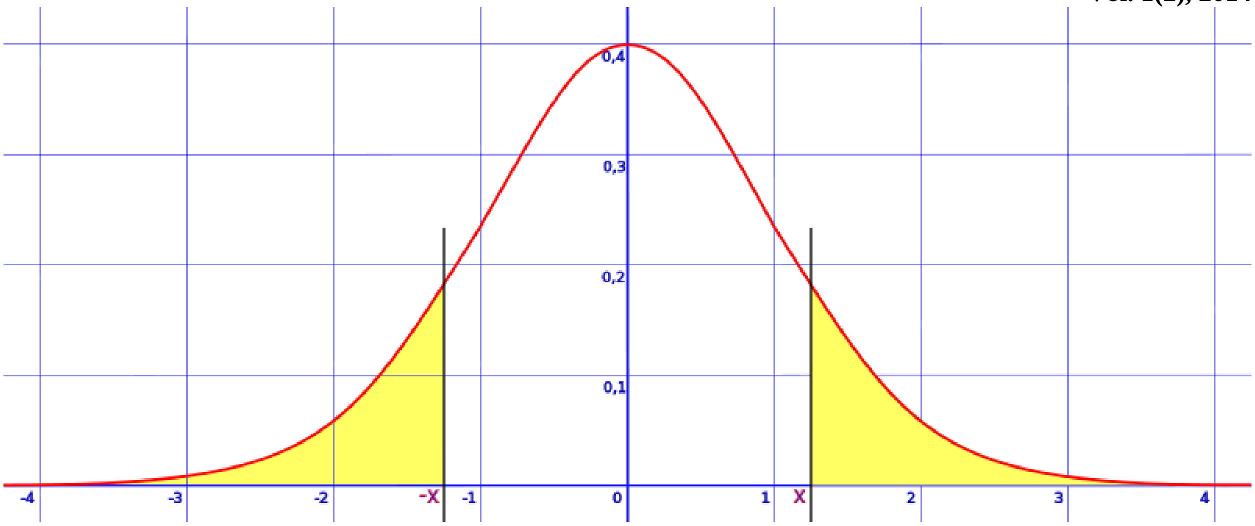
Method of computing statistical significance of data:

One tailed test: It specifies a directional relationship between groups. If we expect a relationship to go one specific way, we are using a one-tailed hypothesis.



Example: If we correlate income with experience, we might hypothesize that income tend to increase with experience. That is a one-tailed hypothesis because it specifies that the correlation must be positive.

Two tailed test:If we make no reference to either direction for the purpose of our research, the two-tailed test is used.



Example :If we were correlating age with the temperature, there is no good reason for expecting that the correlation would be positive or negative. Here, we might just want to find out if there were any relationship at all, and that's a two-tailed hypothesis.

TEST STATISTICS:

Test statistics is the testing of hypothesis. It involves mathematical formulas to determine the likelihood of obtaining sample outcomes if the null hypothesis is true. The value of the test statistic is used to make a decision regarding the null hypothesis based upon the probability of rejecting the null hypothesis which is called level of significance.

Level of significance: While doing hypothesis testing ,if the results found in an analysis do not represent the results that would be obtained from using data involving the entire population from which the sample was derived. To set the criteria for a decision, we state the level of significance.

At different level of significance, the critical values of Z_α

Critical value(Z_α)	Level of significance(α) 1%	5%	10%
Two tailed test	$\lvert Z_\alpha \rvert$=2.58	$\lvert Z_\alpha \rvert$=1.96	$\lvert Z_\alpha \rvert$=1.645

Right tailed test	Z_α =2.58	Z_α =1.64	Z_α =1.28
Left tailed test	Z_α =-2.33	Z_α =-1.645	Z_α =-1.28

Level of confidence: It is a confidence on our judgement that we're not making any mistake while testing the hypothesis .

SAMPLING ERROR IN HYPOTHESIS TESTING

Sampling error: Sampling error is the difference between the value of a statistic and the parameter it estimates. Because of sampling error the value of statistic is not identical to population parameter. Larger the size of sample, lesser the sampling error. Sampling errors are mainly of two types.

Type I error: A Type I error occurs when the researcher reject a null hypothesis when it shouldn't, be rejected then some error occurs known as Type I error.The probability of committing a Type I error is called the significance level. and is often denoted by α. $\{\text{Reject } H_0 \text{ when it is true}\} = P\{\text{Reject } H_0 / H_0\} = \alpha$

Type II error. A Type II error occurs when the researcher fails to reject a null hypothesis that is not true. The probability of committing a Type II error is called Beta, and is often denoted by β. $P\{\text{Accept } H_0 \text{ when it is wrong}\} = P\{\text{Accept } H_0 / H_1\} = \beta$

Type II error and power of test:

Power of the test. The probability of not committing a Type II error is called the power of the test.

TESTING HYPOTHESIS THROUGH VARIOUS PARAMETRIC TESTS AND NON PARAMETRIC TESTS:

Parametric tests are usually more powerful and generally to be preferred. However, parametric tests require certain assumptions to be met in order to be valid. Three common assumptions are:

- **a.** normal distribution of data,
- **b.** interval or ratio data, and
- **c.** randomization of sampling.

If the distribution is extremely skewed, nonparametric tests should be used. Nonparametric tests make no assumptions about the shape of the distribution.

Pamametric tests:

T Test:The t- test is the most powerful parametric test for calculating the significance of a small sample mean. A small sample is generally regarded as one of size n<30. The t testis used to determine whether sample mean is significantly different from population mean. The t test involves forming the ratio of actual observed mean and expected mean. The numerator for a t test is the difference between means, and the denominator is the chance difference that would be expected if the null hypothesis is true .The t test determines whether the observed difference is sufficiently larger than a difference that would be expected by chance. The t value from calculation will be compared with the appropriate t table value (depending upon the probability level and the degrees of freedom). If the calculated t value is equal or larger than the table value, then the null hypothesis is rejected otherwise it is accepted.

Degree of freedom: It is the difference of number of observation and the number of constraints.

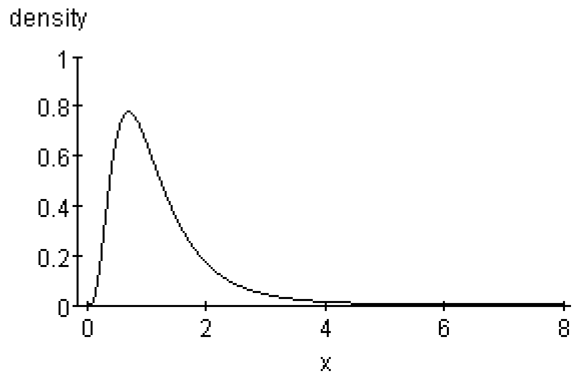
Z TEST: If the sample is large i.e. greater than 30, then statistical theory says that the sample mean is normally distributed then z test can be used. It is a parameter estimation, which is an inference of a sample based on a population of data.

1. Z test for single proportion is used to test a hypothesis on a specific value of the population proportion. Here null hypothesis is H0: p = p0 ,An alternative hypothesis is H1: p > or < p0 where p is the population proportion and p0 is a specific value of the population proportion we would like to test for acceptance.

2. Z test for difference of proportion is used to test the hypothesis that two populations have the same proportion. For example suppose one is interested to test if there is any significant difference in the habit of tea drinking between male and female citizens of a town. In such a situation, Z-test for difference of proportions can be applied. One would have to obtain two independent samples from the town- one from males and the other from females and determine the proportion of tea drinkers in each sample in order to perform this test.

3. Z test for single variance is used to test a hypothesis on a specific value of the population variance. Here the null hypothesis H0: σ = σ0 and Alternative hypothesis H1: σ > or < σ0 where σ is the population mean and σ0 is a specific value of the population variance that we would like to test for acceptance.

F-TEST: The F-distribution is named after the famous statistician R. A. Fisher. The F-test is sensitive to non-normality. F is the ratio of two variances. The F-distribution is most commonly used in Analysis of Variance (ANOVA) and the F test (to determine if two variances are equal). The F-distribution is the ratio of two chi-square distributions, and hence is right skewed. It can never have a negative value. The value can either be 0 or greater than that. The peak of the distribution is not far from 0, as can be seen in the following diagram



A specific F-distribution is denoted by the numerator degrees of freedom (ndf) for the chi-square and the degrees of freedom for the denominator chi-square (ddf), written as F(ndf,ddf). It is important to note that when referencing the F-distribution the numerator degrees of freedom are always given first, and switching the degrees of freedom changes the distribution (ie. F(10,12) does not equal F(12,10)).

Difference between Z-test, F-test, and T-test

Z-test is used for testing the mean of a population against its standard, or comparing the means of two populations, with large samples i.e. where n is greater than or equal to 30, whether you know the population standard deviation or not.

For example: Z test can be used when we want to compare the average daily wages of male and female workers in a factory.

T-test is used for

a.) testing the mean of one population against a standard, or
b.) comparing the means of two populations

If you do not know the populations' standard deviation and when you have a limited sample i.e. less than 30. When the populations' standard deviation is known, z-test is used in that situation.

For example: Measuring the average diameter of big wheel from a certain machine when you have a small sample.

F-test is used to compare 2 populations' variances. The samples can be of any size. It is the basis of ANOVA.

For example: Comparing the variability of bolt diameters from two machines.

Nonparametric testing, Non-parametric tests are used when there are no assumptions made about population distribution –They are also known as distribution free tests, but information is known about sampling distribution. Nonparametric tests can be referred to be a function on a sample that has no dependency on a parameter, whose interpretation does not depend on the population fitting any parameterized distributions. In hypothesis testing, nonparametric tests play a central role for statisticians and decision makers. The application of another justification for the use of nonparametric methods is simplicity. Nonparametric approaches requires when data have a ranking but no clear numerical interpretation.

The chi square goodness of fit test: is based on the difference between the observed and the expected values for each category.

The chi square statistic is defined as $\chi^2 = \sum\sum \frac{(Oij - Eij)^2}{Eij}$

Where Oij is the observed number of cases in category and Eij is the expected number of cases in category. Degee of freedom is one less than the number of categories. The result is a non negative number that tells us how much different the observed values are from the expected values. If we compute that $\chi 2 = 0$, then this indicates that there are no differences between any of our observed and expected values.

FOR EXAMPLE:

A fair coin has a equal probability of half for heads or tails. If we toss a coin 1000 times and record the results of a total of 580 heads and 420 tails. We want to test the hypothesis at 5% level of significance that the toss is fair. We take the null hypothesis H0 is that the toss is fair. Since we are comparing observed frequencies of results from a coin toss to the expected frequencies from an idealized fair coin, a chi-square test should be used.

Calculated value of chi-square. There are two events, heads and tails. Heads has an observed frequency of f1 = 580 with expected frequency of e1 = 500. Tails has an observed frequency of f2 = 420 with expected frequency of e2 = 500.

We now use the formula for the chi-square statistic and see that χ2 = (f1 - e1)2/e1 + (f2 - e2)2/e2= 802/500 + (-80)2/500 = 25.6.

Table value of chi- square:

At degree of freedom 1, χ20.05 =3.841.

Comparison of calculated value with the table value:

Since the calculatedvalue of the chi-square is greater than the table value i.e. 25.6 > 3.841, we reject the null hypothesis and conclude that the toss is not fair.

CONCLUSION:At the end we conclude that a parametric statistical test is one that makes assumptions about the parameters of the population distributions from which data is drawn, as t-tests and the analysis of variance assume the underlying source populations to be normally distributed; they make an assumption that measurement is drawn from an equal-interval scale. while a non-parametric test is one that makes no such assumptions, due to this non-parametric test can be considered as of null category, since all statistical tests assume one thing or another about the properties of the source population of but non-parametric do not make on these particular assumptions.

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